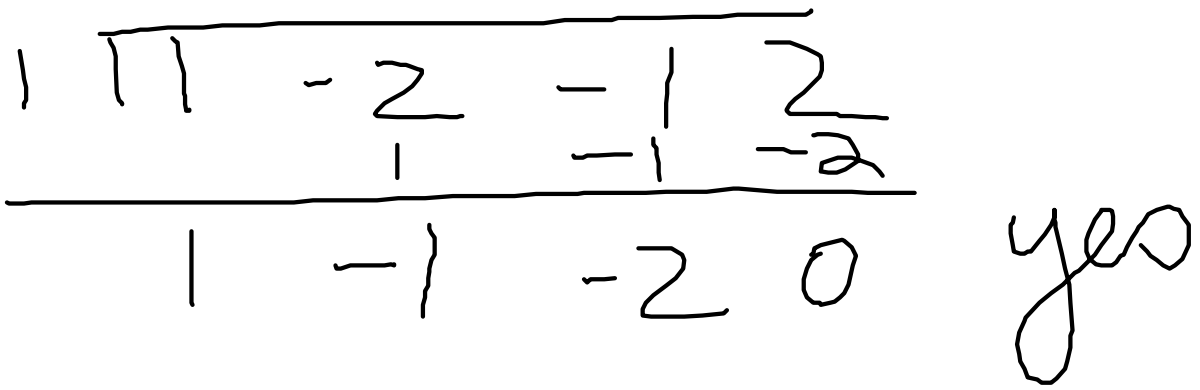
Unit 15
Day 3
Factor Theorem

Factor theorem:

The polynomial x-k is a factor of the polynomial P(x) iff P(k)=0.

Use the factor theorem to decide whether the second polynomial is a factor of the first.

1) $P(x)=x^3-2x^2-x+2;\ x-1$



2) $f(x) = 2x^3 + x^2 + 2; \quad x + 2$

$$\begin{array}{r|rrrr} -2 & 2 & 1 & 0 & 2 \\ & & -4 & 6 & -12 \\ \hline & 2 & -3 & 6 & -10 \end{array}$$

No

Factor f(x) into linear factors given that k is a zero of f(x).

1) $f(x) = 6x^3 + 19x^2 + 2x - 3; \quad -3$

k

$$\begin{array}{r|cccc} -3 & 6 & 19 & 2 & -3 \\ & & -18 & -3 & 3 \\ \hline & 6 & 1 & -1 & 0 \end{array}$$

$f(x) = (x+3)(6x^2 + x - 1)$

$f(x) = (x+3)(3x-1)(2x+1)$

2) $f(x) = 2x^3 + 11x^2 + 7x - 20; \quad -4$

$$\begin{array}{r|rrrr} -4 & 2 & 11 & 7 & -20 \\ & & -8 & -12 & 20 \\ \hline & 2 & 3 & -5 & 0 \end{array}$$

$f(x) = (x+4)(2x^2 + 3x - 5)$

$f(x) = (x+4)(2x+5)(x-1)$

Conjugate zero theorem:

If P(x) is a polynomial having only real coefficients and if $z=a+bi$ is a zero of f(x), where a and b are real numbers, $\overline{z}=a-bi$ is also a zero of f(x).

$\overline{z}$ means the conjugate of z.

Find a polynomial of lowest degree with only real coefficients and having the given zeros. Be sure to simplify the polynomial.

1) 3 + i and 3 - i

$$f(x) = [x-(3+i)][x-(3-i)]$$

$$f(x) = (x-3-i)(x-3+i)$$

$$f(x) = x^2 - 3x + xi - 3x + 9 - 3i - xi + 1 + 3i$$

$$f(x) = x^2 - 6x + 10$$

2) 1 + 2i and 2 (multiplicity 2) $1-2i$

$$P(x) = [x-(1+2i)][x-(1-2i)](x-2)^2$$

$$P(x) = (x^2 - x(1-2i) - x(1+2i) + 5)(x^2-4x+4)$$

$$P(x) = (x^2 - x + 2xi - x - 2xi + 5)(x^2-4x+4)$$

$$P(x) = (x^2 - 2x + 5)(x^2-4x+4)$$

$$P(x) = x^4 - 6x^3 + 17x^2 - 28x + 20$$

Find the polynomial of degree 3 with only real coefficients that satisfies the given condition.

1) zeros of -3, 1 and 4; f(2) = 30

$f(x) = a(x+3)(x-1)(x-4)$

$30 = a(2+3)(2-1)(2-4)$

$30 = a(5)(1)(-2)$

$30 = -10a$

$-3 = a$

$f(x) = -3(x+3)(x-1)(x-4)$

2) zeros of 5, i and -i; and f(2) = 5

HW pg 299-300: 1-14 (all), 36-50 (even)